CS 188: Artificial Intelligence

Review of Search, CSPs, Games

DISCLAIMER: It is insufficient to simply study these slides, they are merely meant as a quick refresher of the high-level ideas covered. You need to study all materials covered in lecture, section, assignments and projects !

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Many slides adapted from Dan Klein

Recap Search I

- Agents that plan ahead → formalization: Search
- Search problem:
  - States (configurations of the world)
  - Successor function: a function from states to lists of (state, action, cost) triples; drawn as a graph
  - Start state and goal test
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)
- Search Algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)

Recap Search II

- Tree Search vs. Graph Search
- Priority queue to store fringe: different priority functions → different search method
  - Uninformed Search Methods
    - Depth-First Search
    - Breadth-First Search
    - Uniform-Cost Search
  - Heuristic Search Methods
    - Greedy Search
    - A* Search --- heuristic design!
      - Admissibility: h(n) <= cost of cheapest path to a goal state. Ensures when goal node is expanded, no other partial plans on fringe could be extended into a cheaper path to a goal state
      - Consistency: c(n->n') >= h(n) – h(n'). Ensures when any node n is expanded during graph search the partial plan that ended in n is the cheapest way to reach n.
- Time and space complexity, completeness, optimality
- Iterative Deepening (great space complexity!)

Reflex Agent | Goal-based Agents

Reflex Agent

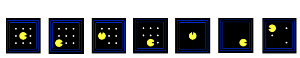
- Choose action based on current percept (and maybe memory)
- May have memory or a model of the world's current state
- Do not consider the future consequences of their actions
- Act on how the world IS
- Can a reflex agent be rational?

Goal-based Agents

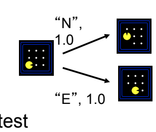
- Plan ahead
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Act on how the world WOULD BE

Search Problems

- A search problem consists of:
  - A state space

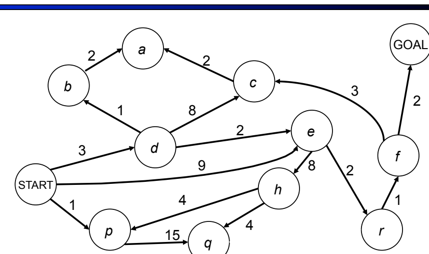


  - A successor function



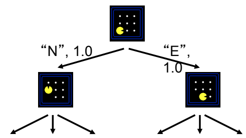
  - A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

Example State Space Graph



Ridiculously tiny search graph for a tiny search problem

Search Trees



- A search tree:
  - This is a "what if" tree of plans and outcomes
  - Start state at the root node
  - Children correspond to successors
  - Nodes contain states, correspond to PLANS to those states
  - For most problems, we can never actually build the whole tree

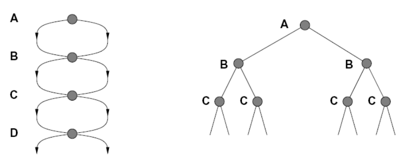
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
  end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work. Why?



Graph Search

- Very simple fix: never expand a state twice

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
     if fringe is empty then return failure
     node ← REMOVE-FRONT(fringe)
     if GOAL-TEST(problem, STATE[node]) then return node
     if STATE[node] is not in closed then
        add STATE[node] to closed
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
  end
```

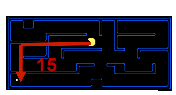
- Can this wreck completeness? Optimality?

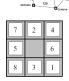
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal
- Often, admissible heuristics are solutions to *relaxed problems,* with new actions ("some cheating") available
- Examples:





- Number of misplaced tiles
- Sum over all misplaced tiles of Manhattan distances to goal positions

Trivial Heuristics, Dominance

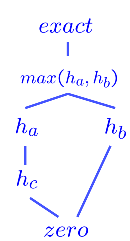
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic



Consistency

- Consistency: $c(n,a,n') \geq h(n) - h(n')$
- Required for A* graph search to be optimal
  - It ensures that when a node gets expanded, that node's final state was reached along the shortest path to reach that final state
- Consistency implies admissibility

A* heuristics --- pacman trying to eat all food pellets

- Consider an algorithm that takes the distance to the closest food pellet, say at (x,y). Then it adds the distance between (x,y) and the closest food pellet to (x,y), and continues this process until no pellets are left, each time calculating the distance from the last pellet. Is this heuristic admissible?
- What if we used the Manhattan distance rather than distance in the maze in the above procedure?

A* heuristics

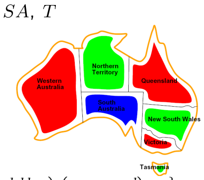
- A particular procedure to quickly find *a* perhaps suboptimal solution to the search problem is in general not admissible.
  - It is only admissible if it always finds the optimal solution (but then it is already solving the problem we care about, hence not that interesting as a heustic).
- A particular procedure to quickly find a perhaps suboptimal solution to a relaxed version of the search problem need not be admissible.
  - It will be admissible if it always finds the *optimal* solution to the relaxed problem.

Recap CSPs

- CSPs are a special kind of search problem:
  - States defined by values of a fixed set of variables
  - Goal test defined by constraints on variable values
- Backtracking = depth-first search (why?, tree or graph search?) with
  - Branching on only one variable per layer in search tree
  - Incremental constraint checks ("Fail fast")
- Heuristics at our points of choice to improve running time:
  - Ordering variables: Minimum Remaining Values and Degree Heuristic
  - Ordering of values: Least Constraining Value
  - Filtering: forward checking, arc consistency
    - → computation of heuristics + pruning of domains might lead to early realization need to backtrack
- Structure: Disconnected and tree-structured CSPs are efficient
  - Non-tree-structured CSP can become tree-structured after some variables have been assigned values
- Iterative improvement: min-conflicts is usually effective in practice

Example: Map-Coloring

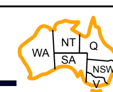
- Variables: $WA,\ NT,\ Q,\ NSW,\ V,\ SA,\ T$
- Domain: $D = \{red, green, blue\}$
- Constraints: adjacent regions must have different colors



  - Implicit: $WA \neq NT$
  - Explicit: $(WA, NT) \in \{(red, green), (red, blue), (green, red), \ldots\}$
- Solutions are assignments satisfying all constraints, e.g.:

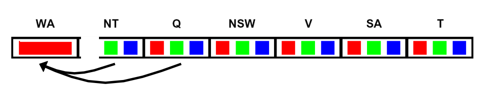
$$\{WA = red, NT = green, Q = red,$$
$$NSW = green, V = red, SA = blue, T = green\}$$

Consistency of An Arc



- An arc X → Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint

Delete from tail!



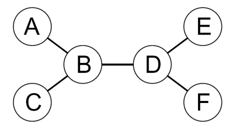
- If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking, but more work!
- Can be run as a preprocessor or after each assignment
- Forward checking = Enforcing consistency of each arc pointing to the new assignment

Backtracking with MRV, Degree, LCV, Filtering

```
function RecursiveBacktracking(pa, fd, vars, constraints)
    if  IsComplete(pa)  then return pa
    next_var <-- select_MRV_Degree(pa, fd, vars, constraints)
    for each value in fd[next_var] do
        new_fd[value] <-- constraint_prop(pa, fd, vars, constraints)
    for each value in fd[next_var] in order of LCV do
        if any of the domains in new_fd[value] is empty
            continue;
        else  // all domains in new_fd[value] have at least one value remaining
            add {var=value} to pa
            result <-- recursive_backtracking(pa, new_fd[value], vars, constraints)
            if (result not equal to failure) then return result
    //if we get here none of the expansions led to a solution
    return failure
```

- select_MRV_degree: selects an unassigned variable based on MRV an degree heuristic
- constraint_prop:performs constraint propagation, this could be through forward propagation or through arc consistency
- pa: partial assignment
- fd: filtered domains

Tree-Structured CSPs



- **Theorem:** if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$
- This property also applies to probabilistic reasoning (later): an important example of the relation between syntactic restrictions and the complexity of reasoning.

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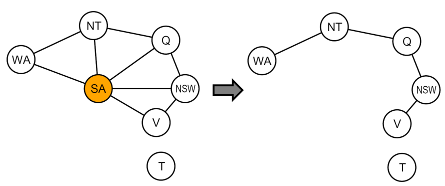
Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c)\ (n-c)\ d^2)$, very fast for small c

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Hill Climbing

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
  - Complete?
  - Optimal?
- What's good about it?

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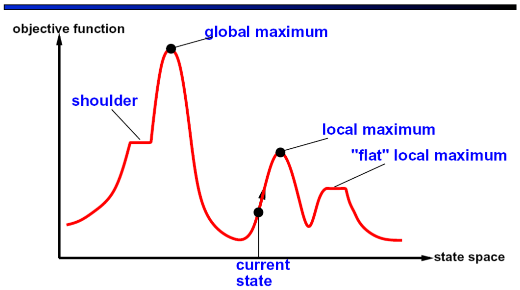
Hill Climbing Diagram



- Random restarts?
- Random sideways steps?

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Recap Games

- Want algorithms for calculating a strategy (policy) which recommends a move in each state
- Deterministic zero-sum games
  - Minimax
  - Alpha-Beta pruning:
    - speed-up up to: $O(b^d) \rightarrow O(b^{d/2})$
    - exact for root (lower nodes could be approximate)
  - Speed-up (suboptimal): Limited depth and evaluation functions
  - Iterative deepening (can help alpha-beta through ordering!)
- Stochastic games
  - Expectimax
- Non-zero-sum games

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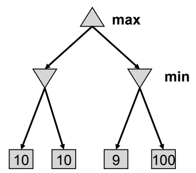
Minimax Properties

- Optimal against a perfect player. Otherwise?
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - O(bm)
- For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



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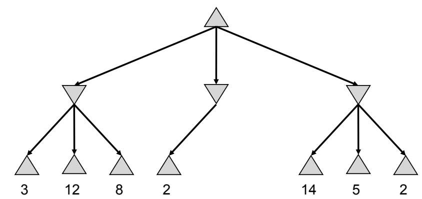
Pruning



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Evaluation Functions

- With depth-limited search
  - Partial plan is returned
  - Only first move of partial plan is executed
  - When again maximizer's turn, run a depth-limited search again and repeat
- How deep to search?

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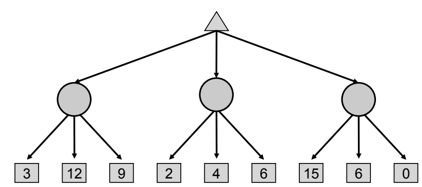
Expectimax



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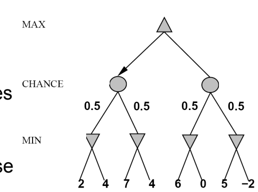
Stochastic Two-Player

- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



if *state* is a MAX node **then**
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

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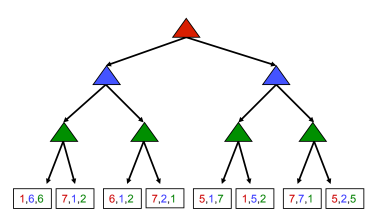
Non-Zero-Sum Utilities

- Similar to minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own utility and propagate (or back up) nodes from children
  - Can give rise to cooperation and competition dynamically…



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